when physically realizable values are considered, it is known that

$$0 < \left(f_{12} - \frac{f_2}{g} \times 10^{n_2-m-n_{12}}\right) < 10. \quad (4)$$

Note that the accuracy of F_1 is dominated by the factor $10^{n_{12}}$ and it is difficult to obtain

$$f_{12} - \frac{f_2}{g} \times 10^{n_2-m-n_{12}}$$

accurately in conventional measuring techniques.

For example, if

$$F_1 = [5 \text{ db}] = 3.1 \times 10^0 \quad (5)$$

$$F_2 = [40 \text{ db}] = 1 \times 10^4 \quad (6)$$

and

$$G_1 = [10 \text{ db}] = 1 \times 10^1 \quad (7)$$

then

$$F_{12} = F_1 + \frac{F_2 - 1}{G_1}$$
$$= [30.013 \text{ db}] \approx [30 \text{ db}] = 1 \times 10^3. \quad (8)$$

However, when (6)–(8) are substituted back into (3)

$$F_1 \approx \left(f_{12} - \frac{f_2}{g} \times 10^{n_2-m-n_{12}}\right) \times 10^{n_{12}}$$
$$= 0 \times 10^3. \quad (9)$$

Thus in practical measurement, it is very difficult to obtain $F_1 = 0.0031 \times 10^3 = 5$ db in this way.

The following new method is proposed to eliminate this kind of problem. According to the definition of noise figure[1,2]

$$F_1 = \frac{N_0}{kT_0B_1G_1} \quad (10)$$

where

N_0 = available noise output of the amplifier under test when the input is terminated by a reflectionless termination,
k = Boltzmann's constant,
T_0 = input noise temperature of the amplifier, and
B_1 = noise bandwidth of the amplifier.

If the auxiliary receiver of gain G_2, noise bandwidth B_2, and noise figure F_2 is connected after the microwave amplifier, the available noise output of the auxiliary receiver N_1 is[1]

$$N_1 = \left(N_0\frac{B_2}{B_1}\right) G_2 + (F_2 - 1)kT_0B_2G_2 \quad (\text{when } B_1 > B_2)$$
$$N_1 = N_0G_2 + (F_2 - 1)kT_0B_2G_2 \quad (\text{when } B_1 < B_2). \quad (11)$$

The available noise output of the auxiliary receiver alone with its input terminated by a reflectionless termination is

$$N_2 = kT_0B_2G_2F_2. \quad (12)$$

Therefore, when $B_1 > B_2$,

$$\frac{N_1}{N_2} \doteq \frac{N_0}{kT_0B_1F_2} + 1 \quad (\because F_2 \gg 1, \text{ in this case})$$

or

$$N_0 = kT_0B_1F_2\left(\frac{N_1}{N_2} - 1\right). \quad (13)$$

Substituting (13) into (10) yields the result,

$$F_1 = \frac{F_2}{G_1}\cdot\left(\frac{N_1}{N_2} - 1\right). \quad (14)$$

For the case of $B_1 < B_2$,

$$\frac{N_1}{N_2} \approx \frac{N_0}{kT_0B_2F_2} + 1$$

or

$$N_0 = kT_0B_2F_2\left(\frac{N_1}{N_2} - 1\right). \quad (15)$$

Substituting (15) into (10) yields the result,

$$F_1 = \frac{F_2}{G_1}\cdot\frac{B_2}{B_1}\cdot\left(\frac{N_1}{N_2} - 1\right). \quad (16)$$

Advantages of this new method are:

1) Since this method does not include a complicated subtraction of measured values as seen in (1) but mostly multiplications and divisions, there is no problem of poor accuracy as shown in (9).
2) This method is simpler than conventional methods as seen from the comparison of (14) with (1).
3) This method does not require measurements of F_{12} which is required in the conventional method.
4) No power measurement is required. The ratio of N_1/N_2 is required which is generally easier than measuring the power. In conventional methods, if the "small signal method" was employed to measure F_{12}, the power measurement is required.

There is a limitation in this new method. This method requires measurements of B_1 and B_2 when $B_1 < B_2$. These measurements are, however, required also in the conventional method when F_{12} and F_2 are measured by the "small signal method." The conventional "noise lamp method" does not require the noise bandwidth measurement.

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[2] N. Houlding, "Noise factor," *Microwave J.*, vol. 5, pp. 74–78; January, 1962.

On Voltage and Current Continuity in the Youla-Weissfloch Circuit*

Any lossless $2n$-port can be represented as a cascade of three $2n$-ports:[1] an all-pass network, a set of n uncoupled ideal transformers, and another all-pass network. The $2n$-port may be either a physical $2n$-port [*i.e.*, a device with an even number of ports considered as a device mapping the scattering (impedance) matrix of a device connected to one half of the ports into the input scattering (impedance) matrix], or it may be a multimode transmission line. In the latter case the network may be a set of uniform lines with a discontinuity, and there may be a condition that the mode voltages or currents are continuous across the discontinuity. If a_1, a_2, b_1, and b_2 are the incident and reflected wave vectors, the voltage continuity requires $a_1+b_1=a_2+b_2$, while current continuity requires $a_1-b_1=a_2-b_2$. If the device is represented by a transfer scattering matrix,

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

the voltage continuity may be stated as:

$$[1 \ 1]\begin{bmatrix} A & B \\ C & D \end{bmatrix} = [1 \ 1]; \quad (1a)$$

where 1 is the $n \times n$ identity matrix.

For current continuity,

$$[-1 \ 1]\begin{bmatrix} A & B \\ C & D \end{bmatrix} = [1 \ -1]. \quad (1b)$$

Eq. (1a) requires $A+C=1$ and $B+D=1$, while (1b) would require that $C-A=1$ and $B-D=1$. The close similarity between voltage and current continuity is noted, and the detailed results that will be developed for voltage continuity will hold for current continuity with some changes of sign.

The transfer scattering matrix of the voltage continuous $2n$-port is written as:

$$T = \begin{bmatrix} A & 1-D \\ 1-A & D \end{bmatrix}. \quad (2)$$

The conditions for a lossless $2n$-port are $T^*\sigma_2T=\sigma_2$, where * is a complex conjugate transpose and

$$\sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

This leads to the following relationships:

$$A^* + A = 2, \quad (3)$$
$$A - D^* = 0, \quad (4)$$
$$D' + \bar{D} = 2. \quad (5)$$

(The primes are transposes and bars signify complex conjugates.)

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[1] D. C. Youla, "Weissfloch equivalents for lossless 2n-ports," IRE TRANS. ON CIRCUIT THEORY, vol. CT-7, pp. 193–199; September, 1960.

If reciprocity is assumed, then $A=\overline{D}$. Eq. (3) implies that $A=1+H_A$, where H_A is an anti (skew)-hermitian matrix. Similarly $D=1+H_A$. Eq. (4) interrelates the two anti-hermitian matrices, $H_A=-\hat{H}_A$. Reciprocity would require $H_A=H_A'$, and thereby H_A would have only a imaginary part.

It is noted that the resultant transfer scattering matrix,

$$T = \begin{bmatrix} 1 + H_A & H_A \\ -H_A & 1 - H_A \end{bmatrix}, \qquad (6)$$

will satisfy the condition for symmetry, *i.e.*, the network can be turned end for end without affecting network performance. This follows[2] since $T\sigma_3 T=\sigma_3$ where

$$\sigma_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Another class of equivalent circuits has been proposed by Felsen, Kahn, and Levey.[3]

[2] W. K. Kahn, "A Theoretical and Experimental Investigation in Multimode Networks and Waveguide Transmission," Microwave Res. Inst., Polytechnic Inst. Brooklyn, Brooklyn, N. Y., Rept. PIBMRI, pp. 818–860; September, 1960.

[3] L. B. Felsen, W. K. Kahn, and L Levey, "Measurement of two-mode discontinuities in a multimode waveguide by a resonance technique," IRE Trans on Microwave Theory and Techniques, vol. MTT-7, pp. 102–110, January, 1959.

This consists of n uncoupled uniform lossless transmission lines with a lossless n-port connected in shunt across the lines (see Fig. 1). If the n-port has an admittance matrix $\hat{Y}$, the transfer scattering matrix is found from the following relations:

$$a_1 + b_1 = a_2 + b_2, \qquad (7)$$

$$a_2 - b_2 + a_1 - b_1 = \hat{Y}(a_2 + b_2). \qquad (8)$$

Then

$$T_{11} = 1 - \hat{Y}/2, \qquad (9a)$$

$$T_{12} = -\hat{Y}/2, \qquad (9b)$$

$$T_{21} = \hat{Y}/2, \qquad (9c)$$

$$T_{22} = 1 + \hat{Y}/2. \qquad (9d)$$

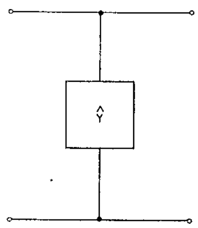

Fig. 1—Shunt representation of a voltage-continuous discontinuity.

$\hat{Y}$ is lossless; hence it is an anti-hermitian matrix. By comparing the relations 6 and 9, one sees that the identification $\hat{Y}=-2H_A$ will specify the Felsen, Kahn, and Levey circuit for any lossless voltage continuous discontinuity. Therefore, it is seen that the two representations are equivalent.

If current continuity were assumed instead of voltage continuity, the only change would be that the matrices A and D would have a skew-hermitian perturbation from the negative of the identity matrix instead of the identity matrix. The Felsen, Kahn, and Levey circuit, however, would have a series discontinuity and $\hat{Y}$ would be replaced by $\hat{Z}$.

In a note,[4] on which this letter is based, the constituents of the Youla-Weissfloch circuit are given explicitly.

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[4] L. J. Kaplan, D. J. R. Stock, and D. C. Youla, "On voltage and current continuity in the Weissfloch circuit," Elec. Engrg. Dept., New York University, N. Y., Tech. Note No. 400–10; May, 1962.